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reciprocal parabola, the directrix of the conic to the center of the reciprocal circle which is on the directrix of the parabola.

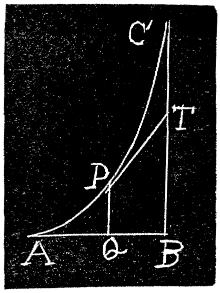
CALCULUS.

314. Proposed by REV. J. H. MEYER, S. J., New Orleans, La.

A fox started from a certain point and ran due east 300 yards, when it was overtaken by a hound that started from a point 100 yards due north of the fox's starting point, and ran directly towards the fox throughout the race. Find the lengh of the curve described by hound, both having started at the same instant, with a uniform velocity.

Solution by J. SCHEFFER, A. M., Hagerstown, Maryland, and FRANCIS E. RUST, E. E., Pittsburg, Pa.

Let A be the starting point of the hound, and B that of the fox, C the point of capture, P some point of the curve described by the hound, $AQ=x$, $PQ=y$, DPT a tangent to the curve at P, $AB=a$ $(=100)$, $BC=b(=300)$; $AP=s$, $m=$rate of hound, $n=$ rate of fox; $s=mt$, $BT=nt$, t being a certain time. $BT=y+(a-x)\tan TBE=y+(a-x)\dfrac{dy}{dx}$.



$\therefore\ \dfrac{s}{m}=\dfrac{y+(a-x)\,(dy/dx)}{n}$; or, putting $\dfrac{n}{m}=\beta$, $\beta s=y+(a-x)\dfrac{dy}{dx}$. Differentiating, $\beta\dfrac{ds}{dx}=(a-x)\dfrac{d^2y}{dx^2}$; or

$\beta\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}=(a-x)\dfrac{d^2y}{dx^2}$. Putting $\dfrac{dy}{dx}=p$, $\beta\sqrt{(1+p^2)}=(a-x)\dfrac{dp}{dx}$.

$\therefore\ \beta\dfrac{dx}{a-x}=\dfrac{dp}{\sqrt{(1+p^2)}}$. $\therefore\dfrac{C_1}{(a-x)^\beta}=p+\sqrt{(1+p^2)}$, and since for $x=0$, $p=0$, $\dfrac{C_1}{a^\beta}=1$, and $C_1=a^\beta$. Now, $\dfrac{a^\beta}{(a-x)^\beta}=p+\sqrt{(1+p^2)}$, or $a^\beta(a-x)^{-\beta}=p+\sqrt{(1+p^2)}$; whence $p=\dfrac{dy}{dx}=\frac{1}{2}[a^\beta(a-x)^{-\beta}-a^{-\beta}(a-x)^\beta]$.

$\therefore y=\dfrac{a^{-\beta}(a-x)^{1+\beta}}{1+\beta}-\dfrac{a^\beta(a-x)^{1-\beta}}{1-\beta}+C_2$; but $0=\frac{1}{2}\left(\dfrac{a^{-\beta}a^{1+\beta}}{1+\beta}-\dfrac{a^\beta a^{1-\beta}}{1-\beta}\right)+C_2$, whence $C_2=\dfrac{a^\beta}{1-\beta^2}$.

$\therefore y=\frac{1}{2}\left[\dfrac{a^{-\beta}(a-x)^{1+\beta}}{1+\beta}-\dfrac{a^\beta(a-x)^{1-\beta}}{1-\beta}\right]+\dfrac{a^\beta}{1-\beta^2}$; $\dfrac{ds}{dx}=\frac{1}{2}[a^\beta(a-x)^{-\beta}+a^{-\beta}(a-x)^\beta]$;

$$\therefore s=-\tfrac{1}{2}\left[\frac{a^{\beta}\,(a-x)^{1-\beta}}{1-\beta}+\frac{a^{-\beta}\,(a-x)^{1+\beta}}{1+\beta}\right]+C_3.\quad \text{For } x=0,\ s=0.$$

$$\therefore C_3=\frac{a}{1-\beta^2}.\quad \therefore s=\frac{a}{1-\beta^2}-\tfrac{1}{2}\left[\frac{a^{\beta}\,(a-x)^{1-\beta}}{1-\beta}+\frac{a^{-\beta}\,(a-x)^{1+\beta}}{1+\beta}\right].$$

Since for $x=a$, $y=b$, we have $b=\frac{a^{\beta}}{1-\beta^2}$; or, substituting $a=100$, $b=300$, we have $\beta^2+\frac{1}{3}\beta=1$; whence $\beta=\frac{1}{6}(\sqrt{37}-1)$ and length of curve between A and $C=\frac{a}{1-\beta^2}=50(\sqrt{37}+1)=354.135$.

$$y=\frac{na}{n^2-1}-\frac{na^{1/n}}{2(n-1)}(a-x)^{(n-1)/n}+\frac{n}{2(n+1)}\,\frac{1}{a^n}(a-x)^{(n-1)/n}.$$

315. Proposed by C. N. SCHMALL, New York City.

If $y=f(x)$, show by Taylor's Theorem that

$$f\left(\frac{x}{1+x}\right)=y-\frac{x^2}{1+x}\cdot\frac{dy}{dx}+\frac{x^4}{2\,(1+x)^2}\cdot\frac{d^2y}{dx^2}-\frac{x^6}{2.3.(1+x)^3}\cdot\frac{d^3y}{dx^3}+\dots\text{ etc.}$$

Solution by the PROPOSER.

Put $\frac{x}{1+x}=x+h$, then $f\left(\frac{x}{1+x}\right)=f(x+h)$,

and also, $h=\frac{x}{1+x}-x=-\frac{x^2}{1+x}$.

$$\therefore h^2=\frac{x^4}{(1+x)^2},\quad h^3=-\frac{x^6}{(1+x)^3},\ \text{and so on.}$$

Substituting these values of the powers of h in Taylor's series, we have the required result; *i. e.*, from

$$f(x+h)=f(x)+hf'(x)+\frac{h^2}{2!}f''(x)+\frac{h^3}{3!}f'''(x)+\text{etc.},$$

we get, by direct substitution,

$$f\left(\frac{x}{1+x}\right)=y-\frac{x^2}{1+x}\,\frac{dy}{dx}+\frac{x^4}{2\,(1+x)^2}\,\frac{d^2y}{dx^2}-\frac{x^6}{2.3(1+x)^3}\,\frac{d^3y}{dx^3}+\text{etc.}$$

316. Proposed by C. N. SCHMALL, New York City.

$$\int_0^{\infty}\frac{\cos ax}{1+x^2}dx=\tfrac{1}{2}\,\pi\,e^{-a}=\int_0^{\infty}\frac{x\sin ax}{1+x^2}dx.$$

(From Bromwich, *Theory of Infinite Series*, p. 442, ex. 5, and also from Carslaw, *Fourier's Series*, p. 113, ex. 12.) Prove this by any method.